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**Question Paper Code : 40874**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2025.

Fourth Semester

Computer Science and Engineering

MA 8402 – PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

(Statistical tables may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If  $A$  and  $B$  are mutually exclusive (assuming neither  $A$  nor  $B$  has zero probability) will they be independent? Justify your answer.
2. A lake contains 200 trout; 50 of them are caught randomly, tagged, and returned. If, again, we catch 50 trout at random, what is the probability of getting exactly five tagged trout?
3. Let  $X$  is a random variable. Is the statement  $X + X = 2X$  true? Justify your answer.
4. In the experiment of rolling a balanced die twice, let  $X$  be the maximum of the two numbers obtained. Determine the distribution function of  $X$ .
5. A point is selected at random from the rectangle  $R = \{(x, y) \in R^2 : 0 < x < a, 0 < y < b\}$ . Let  $X$  be x-coordinate and  $Y$  be y-coordinate of the point selected. Determine if  $X$  and  $Y$  are independent random variables.
6. Construct the transition probability matrix(tpm) : “A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is as likely to drive again as he is to travel by train. Now suppose that on the first day of the week. Now suppose that on the first day of the week, the man tossed a fair dice and drive to work if and only if a six appeared”
7. Comment on the statement: “Sum of two independent Poisson processes is also Poisson process”.

8. A fair die is tossed repeatedly. The maximum of the first  $n$  outcomes is denoted by  $X_n$ . Is  $\{X_n; n = 1, 2, \dots\}$  a Markov chain? Justify your answer

9. The following is the transition probability matrix of a Markov chain with state space  $\{0, 1, 2\}$ . Specify the classes, and determine which classes are recurrent.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

10. A tiger is always in one of three states (0) sleeping, (1) hunting and (2) eating. A tiger's life is fairly monotonous, and it always goes from sleeping to hunting to eating and back to sleeping. On average, the tiger sleeps for 3 hours, hunts for 2 hours, and eats for 30 minutes. Assuming that the tiger remains in each state for an exponential time, model the tiger's life by continuous time Markov chain.

#### PART B — (5 × 16 = 80 marks)

11. (a) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:  $P(D|P1) = 0.01$ ,  $P(D|P2) = 0.03$ ,  $P(D|P3) = 0.02$  where  $P(D|Pj)$  is the probability of a defective product, given plan  $j$ . If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Or

(b) Suppose that a system contains a certain type of component whose time, in years, to failure is given by  $T$ . The random variable  $T$  is modeled nicely by the exponential distribution with mean time to failure  $\beta = 5$ . If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years?

12. (a) Let  $X_1$  and  $X_2$  be two independent random variables having Poisson distributions with parameters,  $\mu_1$  and  $\mu_2$ , respectively. Find the distribution of the random variable  $Y_1 = X_1 + X_2$ .

Or

- (b) Let  $X_1$  and  $X_2$  be two continuous random variables with joint probability distribution

$$f(x_1, x_2) = \begin{cases} 4x_1x_2; & 0 < x_1 < 1; 0 < x_2 < 1 \\ 0: & \text{otherwise} \end{cases}$$

Find the joint probability distribution of  $Y_1 = X_1^2$  and  $Y_2 = X_1X_2$

13. (a) Let  $\{N(t): t \geq 0\}$  be a Poisson process with parameter  $\lambda$ . Suppose that for a fixed  $t > 0$ ,  $N(t) = n$ . That is, we are given that  $n$  events have occurred by time  $t$ . For  $u$ ,  $0 < u < 1$ , show that the number of events that have occurred at or prior to  $u$  is binomial with parameters  $n$  and  $u/t$ .

Or

- (b) Mr. Gorfin is a movie buff who watches movies regularly. His son has observed that whether Mr. Gorfin watches a drama or not depends on the previous two movies he has watched with the following probabilities:  $7/8$  if the last two movies he watched were both dramas,  $1/2$  if exactly one of them was a drama, and  $1/8$  if none of them was a drama.
- (i) Mr. Gorfin watched four movies last weekend. If the first two were dramas, what is the probability that the fourth one was a drama as well?
- (ii) What is the long-run probability that Mr. Gorfin will watch two dramas in a row?
14. (a) Construct the state transition diagram of birth-death process and also derive the steady state distribution of birth-death process.

Or

- (b) A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs in order in which they come in and if the arrivals of repaired system follows Poisson process with an arrival rate 10 per 8 hour day, what is the repair man's idle time each day? How many jobs are ahead in the system while new repaired system arrives?
15. (a) Derive Pollaczek Khinchin formula for M/G/1 queueing system.

Or

- (b) Define PASTA property in queueing models. How is it useful in queueing networks.

